(Don't fear) the repo. A simple model to save regional banks.

Asset allocation

There are three periods. At *t-1* a bank receives deposits and allocates its portfolio between bills and coupon bonds; the bill matures at *t* and the bond at *t+1*. The interest rate the next period is known, the interest rate at *t+1* is based on expectations. At time *t*, the interest rate at time *t+1* is revealed, and some (or all) deposits leave the bank. Whether this renders the bank insolvent or not is something we wish to explore.

Portfolio allocation at *t-1* is between bills (B_t) and the bond (B_{t+1}) with coupon (C_{t+1}) falling in both the period. Using the usual bond pricing formula (discounting) assets allocated at *t-1* are:

$$A_{t-1} = \frac{B_t}{1+i_t} + \left(\frac{C_{t+1}}{1+i_t} + \frac{C_{t+1} + B_{t+1}}{(1+i_t)(1+i_{t+1}^e)}\right) = \frac{B_t}{1+i_t} + \frac{B_{t+1}}{1+i_t}\left(c_{t+1} + \frac{1+c_{t+1}}{1+i_{t+1}^e}\right)$$

$$= \frac{B_t}{1+i_t} + \frac{B_{t+1}}{1+i_t}(1+c_{t+1}) = B_t\left(\frac{1+(1+c_{t+1})\varepsilon}{1+i_t}\right)$$

where $c_{t+1} = C_{t+1}/B_{t+1}$ is the coupon yield, $\varepsilon = B_{t+1}/B_t$ the ratio of duration assets to bills in the bank balance sheet as chosen at *t-1* (before the interest rate at *t+1* is revealed) and it is assumed the coupon yield is equal to the interest rate expected at *t+1* ($c_{t+1} = i_{t+1}^e$)

We normalize here by dividing by the assets at *t-1* making lower-case letter the ratio of the assets or liabilities-to-total assets:

$$1 = \frac{b_t}{1+i_t} + \frac{b_{t+1}}{1+i_t}(1+c_{t+1}) = b_t\left(\frac{1+(1+c_{t+1})\varepsilon}{1+i_t}\right)$$

and we can rearrange to write the bill holdings as:

$$b_t = \left(\frac{1+i_t}{1+\varepsilon(1+c_{t+1})}\right)$$

whereas:

$$b_{t+1} = \left(\frac{1+i_t}{1/\varepsilon + (1+c_{t+1})}\right)$$

such that ε is a simple measure of the portfolio choice between bills and bonds; $\varepsilon \to \infty$ meaning asset holdings are concentrated exclusively in bonds, whereas $\varepsilon = 0$ only in bills and $\varepsilon = 1$ the portfolio is split evenly between the two.

Liabilities are given by $1 = m_t + m_{t+1}$; it is not known at the time of the allocation whether deposits are withdrawn at *t* or *t+1* creating a risk to banks when they allocate assets.

Period t cash flow

During period t, the bank's cash flow is given by the interest on bills and the coupon on the bond, plus the remaining face value of the bill minus deposits withdrawn:

$$Cash\ flow = \frac{i_t b_t}{1+i_t} + c_{t+1} b_{t+1} + \frac{b_t}{1+i_t} - (1+i_t^D) m_t$$

notice how the deposit rate (i_t^D) could differ from the one-period interest on bills (i_t). This simplifies as:

$$Cash\ flow = (1+\varepsilon c_{t+1}) b_t - (1+i_t^D) m_t = (1+\varepsilon c_{t+1}) b_t - (1+i_t^D) + (1+i_t^D) m_{t+1}$$

where the right side substitutes an expression for$m_t = 1 - m_{t+1}$ so that the deposits rolled into period *t+1* (including interest) can be thought of as a positive cash flow. It is as if the deposits are withdrawn in full and some deposited again (with interest from the first period).

Substituting in our expression for b_t we get:

$$Cash\ flow = \frac{(i_t - i_t^D)(1+\varepsilon c_{t+1}) - (1+i_t^D)\varepsilon}{1+\varepsilon(1+c_{t+1})} + (1+i_t^D) m_{t+1}$$

We assume no income for equity holders of the bank in cash at time *t*; profits (if they exist) are withdrawn only at time *t+1*.

If the bank has no duration assets, where $\varepsilon = 0$, they make a pick-up on holding bills over deposits ($i_t > i_t^D$) as compensation for intermediation services, plus the money liabilities rolled into the next period. But if $\varepsilon > 0$ the bank makes some coupon income as well but has *no maturing bond principal* so could be found illiquid—in the sense of needing to sell their bond portfolio if enough deposits are withdrawn.

Period t portfolio allocation

If outflows are large enough, the bank will sell their bond portfolio (that might otherwise be HTM) at time *t* to meet liquidity needs. Assume no other assets to carry from period *t* to *t+1*. Let $\bar{b}_{t+1}$ denote bond remaining after allowing for the period *t* cash flow (positive or negative).

Given the interest rate is now known, the bond is now priced above or below par as the actual interest rate deviates from the coupon yield (which is assumed equal to the previous period expectation of the interest rate.) This can be the source of mark to market losses.

$$\frac{1+c_{t+1}}{1+i_{t+1}} \bar{b}_{t+1} = \frac{1+c_{t+1}}{1+i_{t+1}} b_{t+1} + cash\ flow$$

If the cash flow is positive, the bank accumulates more of the bond (now a one-period coupon security), if negative they sell down some of their bond portfolio from *t-1*. Substituting in for cash flow, and the definition of b_{t+1} we get:

$$\bar{b}_{t+1} = \varepsilon b_t + \frac{1+i_{t+1}}{1+c_{t+1}}\left((1+\varepsilon c_{t+1})b_t - (1+i_t^D) + (1+i_t^D)m_{t+1}\right)$$

Solvency constraint

Ultimately, solvency at *t+1* is what matters. We have assumed no value is taken from the bank at time *t* only at time *t+1* when the profit is realized.

The profit (z_{t+1}) of the bank at *t+1* is given by the coupon on the remaining bond portfolio less the interest on deposits carried over:

$$z_{t+1} = (1+c_{t+1})\bar{b}_{t+1} - (1+i_t^D)(1+i_{t+1}^D)m_{t+1}$$

after some manipulation, this can be written as:

$$z_{t+1} = \frac{(1+i_t)(i_{t+1}^e - i_{t+1})\varepsilon}{1+\varepsilon(1+i_{t+1}^e)} + (1+i_{t+1})(i_t - i_t^D) + (1+i_t^D)(i_{t+1} - i_{t+1}^D)m_{t+1}$$

where we have again used the fact that the coupon yield was equal to the expected interest rate at *t+1* here.

The profit of the bank is given by three terms.

- The **first** is the repricing of the bond portfolio due to the interest rate increase at time *t* for *t+1*; as the interest rate increases above the expected interest/coupon yield, the bond portfolio makes both mark-to-market and eventually realized losses—and the larger is the bond portfolio, ε, the larger is this loss. This is $(i_{t+1}^e - i_{t+1})b_{t+1}$.

- **Second**, against this, the bank makes a profit on intermediating the entire stock of deposits for the first period measured in time *t+1* cash given by $(1+i_{t+1})(i_t - i_t^D)$.

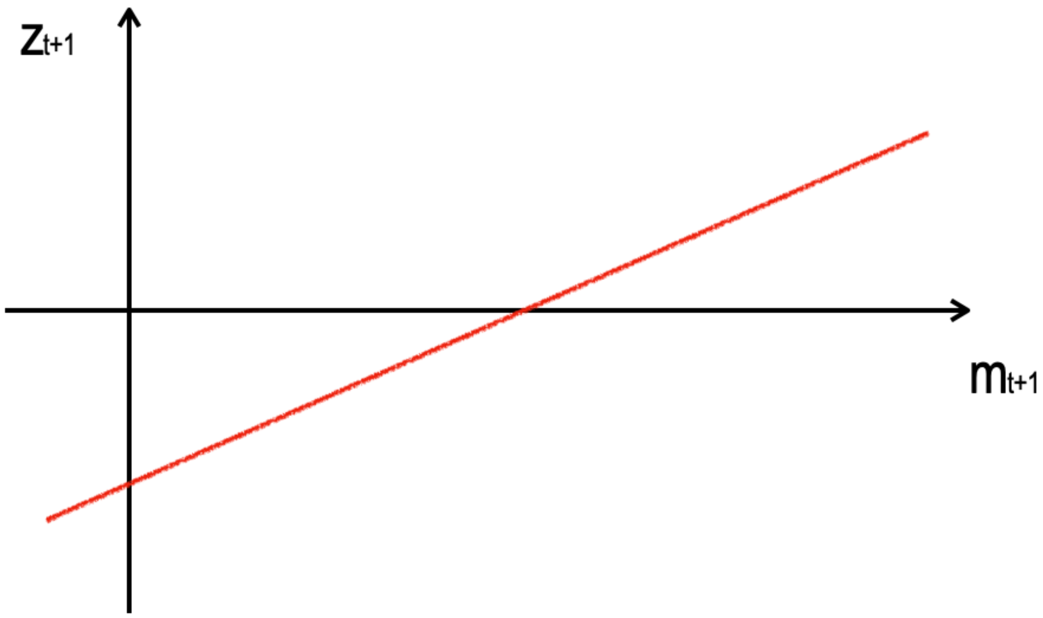
- And **third** there is also a profit from intermediating the deposits carried over into the second period before they are finally withdrawn ($(1+i_t^D)(i_{t+1} - i_{t+1}^D)m_{t+1}$).

Notice how if deposits pay interest at the bill yield throughout ($i_t^D = i_t$ and $i_{t+1}^D = i_{t+1}$), then the profitability of the bank depends only on the bond portfolio. This is a version of the Modigliani-Miller irrelevance theorem—if deposits are priced “correctly” according to the rate available on one-period bills then financing structure is irrelevant. The only thing that matters is the additional coupon pick-up from the bond portfolio. And in a world of secularly declining rates, holding duration is an eminently profitable business indeed. But if the interest rate increases, losses on the bond portfolio will become painful. Such losses can be offset by the pick-up on intermediation services as the deposit rate is below the one-period rate on securities.

If there is no bond portfolio, of course, the bank makes no profit or loss on duration and will only make the intermediation pick-up.

But if they hold a bond portfolio, they make a loss on this portfolio when the interest rate rises. This can be either mark-to-market at *t* as they are sold to provide liquidity or actual losses in period *t+1*. But providing not all deposits leave in period *t*, they have a chance of clawing this back and turning a profit in period *t+1* because the deposit rate is below the one-period bill rate.

In fact, we can plot the period *t+1* profit of the bank as a function of the rolled deposits assuming $i_{t+1} > i_{t+1}^{e}$:



There is a loss due to the realization of interest rate risk if enough deposits are withdrawn immediately, hence the profit function crosses below the x-axis given by losses on the bond portfolio. The larger the bond portfolio and the higher the interest rate, the more negative does this cross the y-axis. But if enough deposits are carried into period t+1, despite these mark-to-market losses the bank *can still be solvent* due to the interest income on deposits.

Or we can write the deposits that need to be retained to achieve zero profit, which is increasing in the portfolio allocated to bonds and an increasing convex function of the interest rate at *t+1*.

$$m_{t+1} = \left(\frac{i_{t+1} - i_{t+1}^{e}}{i_{t+1} - i_{t+1}^{D}}\right)\frac{b_{t+1}}{(1 + i_{t}^{D})}$$

The larger the bond portfolio, the more important it is to have sticky deposits that allow you to earn intermediation income; the larger the interest rate adjustment the more deposits you need to retain.

Enter the repo

How can the central bank's repo facility support banks? After all, they don't need liquidity if they can sell the government bonds easily enough.

Let's imagine that instead of selling their bond portfolio, the banks can access central bank liquidity support to finance the cash outflow in period *t*.

The bond portfolio is unchanged, but repo is accessed to cover the cash flow shortfall

$$r_{t+1} = -Cash\ flow = (1 + i_t^D) - (1 + \varepsilon c_{t+1})b_t - (1 + i_t^D)m_{t+1} \leq hb_{t+1}$$

When cash flow is negative, the period *t+1* profit now becomes:

$$\bar{z}_{t+1} = (1 + c_{t+1})b_{t+1} - (1 + i_{t+1}^R)r_{t+1} - (1 + i_t^D)(1 + i_{t+1}^D)m_{t+1}$$

Which, after some manipulation, turns out to be identical to the above profit equation, but the market interest rate at *t+1* is replaced throughout by the repo rate:

$$z_{t+1}^R = \frac{(1 + i_t)(i_{t+1}^e - i_{t+1}^R)\varepsilon}{1 + \varepsilon(1 + i_{t+1}^e)} + (1 + i_{t+1}^R)(i_t - i_t^D) + (1 + i_t^D)(i_{t+1}^R - i_{t+1}^D)m_{t+1}$$

So the repo serves to blunt the impact of the policy normalization on bank profitability, but can only meaningfully do so if made available at concessional rates. Recall how the Eurozone provided concessional TLTROs (eventually) if certain lending targets were met. This is a sly way of making concessional financing available for the banks.

The difference between the period *t+1* profit under repo support and otherwise is:

$$z_{t+1}^R - z_{t+1} = \frac{(1 + i_t)(i_{t+1} - i_{t+1}^R)\varepsilon}{1 + \varepsilon(1 + i_{t+1}^e)} + (i_{t+1}^R - i_{t+1})(i_t - i_t^D) + (1 + i_t^D)(i_{t+1}^R - i_{t+1})m_{t+1}$$

Suppose the first period deposit rate is equal to the market rate ($i_t = i_t^D$) then providing the repo rate is below the market rate ($i_{t+1} - i_{t+1}^R > 0$) the bank makes a smaller loss on the bond portfolio, possibly a profit, by being able to financing outflows from the repo instead of selling bonds. But they are worse off the larger is the cash rolled into t+1 by depositors (which will not benefit from the repo).

Policy implications

To focus ideas, let's suppose the initial deposit rate is equal to the policy rate $i_t - i_t^D$.

What does this thought experiment tell us?

There are three policy options are available to "save the banks."

- The first is simply to *encourage deposits to remain in the bank* to period *t+1* so there is no need to sell coupon securities (at a loss) while the bank can then benefit from the spread between the second period interest rate and the deposit rate—when the deposit rate is below the one-period rate on assets.
- The second is to *offer repo funding to the banks*. But for this to be successful it has to be made at below market rate to offset some of the cost of selling coupon securities at a loss. In fact, if the repo rate is equal to the expected interest rate at *t+1* (in this example) then losses on their coupon portfolio can be fully offset (and the losses socialized via the central bank balance sheet.)
- The third is to *keep interest rates as low as expected* when the coupon securities were bought during the allocation of assets at *t-1*.

The first imposes the losses from higher rates on depositors who get paid below market rate. The second imposes the costs "on the taxpayer" via the central bank balance sheet. The last is a form of financial dominance, where the inflation target is suppressed in favour of financial stability—and the costs passed on through inflation.

Someone has to pay the price of higher inflation when there are coupon securities issued below the market rate.

Who will it be?