Inside the New Keynesian black box

The New Keynesian (NK) model has apparently become the workhorse for many central banks in setting policy. But it is largely a black box.

To help understand how the basic NK model might be guiding policy right now we walk through a simple version (drawing on Gali) while recognising that the actual models—specific to each central bank—will contain many more bells and whistles. While imperfect, this simple exercise provides at least some insight the forecasting challenge at this time.

Basic New Keynesian model

In its simplest form, the NK model consists of three questions. We assume consumption in utility takes logarithmic form and that the policy rule responds only to inflation and not the output gap. We have:

Dynamic IS equation:

$$\hat{y}_t = E_t\{\hat{y}_{t+1}\} - (i_t - E_t\{\pi_{t+1}\} - r_t^n) \qquad (1)$$

New Keynesian Phillips curve (given Calvo pricing):

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \hat{y}_t \qquad (2)$$

And a Taylor Rule:

$$i_t = \phi_\pi \pi_t \qquad (3)$$

AS-AD framework:

Combining (3) and (1) we get an AD relation, downward-sloping function of the output gap:

$$\pi_t = \frac{1}{\phi_\pi}(\sigma E_t\{\hat{y}_{t+1}\} + E_t\{\pi_{t+1}\} + r_t^n) - \frac{1}{\phi_\pi}\hat{y}_t$$

Iterating (2) forward one period, we get the NKPC as a function of the output gap at *t+1* as expected today and inflation in two periods time:

$$\pi_t = \kappa \hat{y}_t + \kappa\beta E_t\{\hat{y}_{t+1}\} + \beta^2 E_t\{\pi_{t+2}\}$$

Generating an inflation shock

Assume we are in a liquidity trap where the natural rate is zero ($r_t^n = 0$) as are future inflation expectations $E_t\{\pi_{t+1}\} = E_t\{\pi_{t+2}\} = 0$. Given this backdrop, we also suppose—as has been the case—policy is committed *not to respond to inflation* immediately, $\phi_\pi = 0$.

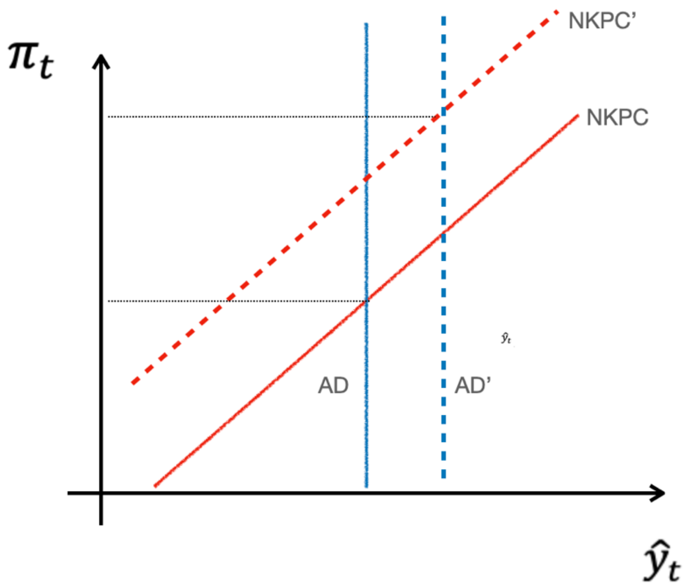
How do we get an initial inflation shock? Well, this can *only* come about in this stripped-down NK model due to an increase the output gap expected in the next period ($E_t\{\hat{y}_{t+1}\}$). In particular, imposing our liquidity trap assumptions, AD no longer depends on π_t but on the next period output gap:

$\hat{y}_t = E_t\{\hat{y}_{t+1}\}$

while AS depends on the current and expected output gap:

$\pi_t = \kappa\hat{y}_t + \kappa\beta E_t\{\hat{y}_{t+1}\}$

The AS-AD set-up is shown diagrammatically:



The AD curve is vertical due to there being no policy feedback in the liquidity trap. It shifts right with the expected future output gap. The AS curve slopes up and likewise shifts up my some multiple of the same expected output gap.

In other words, expectations of a future boom in spending drives an increase in demand today—forward-looking agents smooth consumption, accelerating spending; those able to change prices do so by adjusting them upwards due to a positive output gap today and the prospect of such a gap tomorrow, generating an acceleration in inflation.

Crucially, the action in terms of inflation and the output gap are entirely due to expectations of future activity outstripping potential. Solving for each:

$$\hat{y}_t = E_t\{\hat{y}_{t+1}\}$$

$$\pi_t = \kappa(1+\beta)E_t\{\hat{y}_{t+1}\}$$

The impact on both inflation and output gap in the first period, due to the lack of policy reaction, is unambiguously positive—as is evident also in the diagram above.

Today's output gap is exactly equal to the output gap expected tomorrow. The size of the inflation response depends on the parameter combination $\kappa(1+\beta)$ which may or may not be larger than unity—but providing the Phillips Curve is sufficiently upward sloping, which we are willing to suppose it is, inflation will indeed outstrip the output gap.

With this, we have here our first clue as to the failure across central banks to anticipate the acceleration in the post-pandemic recovery.

To generate inflation in this basic NK model, policymakers *need to be able to anticipate and project a future positive output gap*. Failure to do so means there will be no forecast of accelerating inflation. But the output gap itself is not only unobserved, but also an *ex-post rationalisation*.

The only way we surmise there is a positive output gap *today* is (typically) due to an acceleration in inflation—a tell-tale sign that there is an imbalance between contemporary aggregate supply and demand.

It would seem to be almost impossible for central banks to ever project accelerating inflation in such a world. That is, at least, one way to summarize the challenge when applying the basic New Keynesian model in the current circumstances. We return to this shortly.

Escaping the liquidity trap: Adjusting to higher inflation expectations

Whereas in the first period the economy was considered in a liquidity trap, in the second period higher inflation expectations allow for the normalization of interest rate policy.

This complicates the picture slightly.

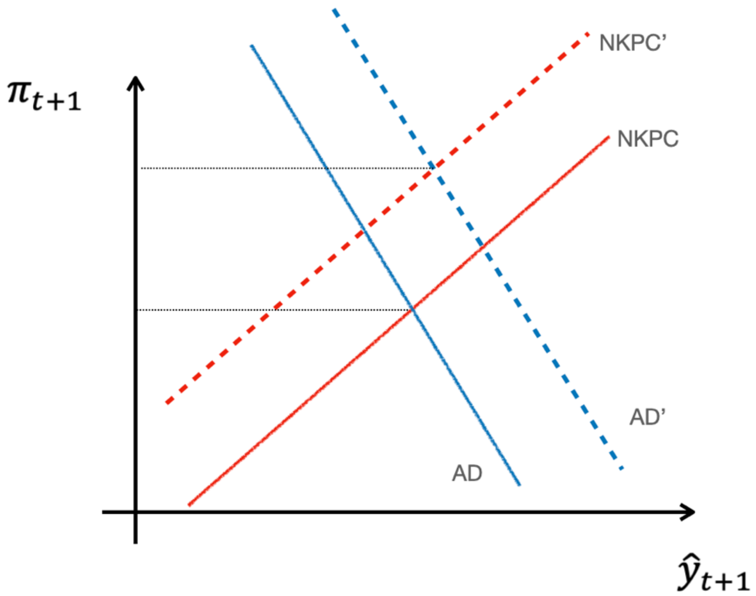
In particular, suppose future inflation expectations $E_t\{\pi_{t+1}\} = E_t\{\pi_{t+2}\} = \pi^e > 0$ while $r^n_{t+1} = r^n$. Then AD at time *t+1* becomes:

$$\pi_{t+1} = \frac{1}{\phi_\pi}(\sigma E_{t+1}\{\hat{y}_{t+2}\} + \pi^e + r^n) - \frac{1}{\phi_\pi}\hat{y}_{t+1}$$

While the AS (NKPC) curve becomes:

$$\pi_{t+1} = \kappa\hat{y}_{t+1} + \kappa\beta E_{t+1}\{\hat{y}_{t+2}\} + \beta^2\pi^e$$

Plotted once again:



In period *t+1* the AD curve is now downward sloping and shifts up due to the increase in inflation expectations, the natural rate, and (potentially) due to the expected output gap in the next period. The AS curve also shifts up (a reduction in supply) on expected inflation as well as the expected (positive) output gap.

So, the second period, upon escaping the liquidity trap, will also see inflation above the pre-recovery baseline if the output gap remains positive. Once again solving:

$$\hat{y}_{t+1} = \frac{1 - \phi_\pi \beta^2}{\kappa \phi_\pi + 1}(\pi^e + r^n) + \frac{1 - \kappa \phi_\pi \beta}{\kappa \phi_\pi + 1} E_{t+1}\{\hat{y}_{t+2}\}$$

$$\pi_{t+1} = \frac{\kappa + \beta^2}{\kappa \phi_\pi + 1}(\pi^e + r^n) + \frac{\kappa(1 + \beta)}{\kappa \phi_\pi + 1} E_{t+1}\{\hat{y}_{t+2}\}$$

The impact on inflation in period *t+1* is positive due to inflation expectations and the natural rate, $\pi^e + r^n$, as agents revise up pricing in anticipation of future prospects; it will also be higher in the agents see a positive output gap continuing in period *t+2* ($E_{t+1}\{\hat{y}_{t+2}\} > 0$). However, if this expected output gap were sufficiently negative, due to anticipation of recession, then inflation this period would be tempered or could start to fall already as forward-looking price setting factors in such bleak prospects.

Likewise, the output gap today depends on $\pi^e + r^n$ as well as $E_{t+1}\{\hat{y}_{t+2}\}$. But the sign on each is now, unlike in the first period, ambiguous *depending on the size of the policy response* to inflation. A sufficiently large policy response, through ϕ_π, would generate a negative output gap today as consumption is postponed due to a greater real interest rate.

From the perspective of practical policymaking and forecasting, inflation once more depends upon the expected output gap as well as expected inflation and the natural rate. All three are unobserved, subject to uncertainty and measurement challenges, and none are directly linked to aggregate spending decisions *today*.

Put another way, the New Keynesian model has shifted the *art* of forecasting macroeconomic variables from macro-financial variables we can observe in close to real time—bank credit, money stocks, private sector net worth, investment, fiscal balance, consumption, net exports, wage growth—to variables that are poorly understood and often only estimated with any confidence after the fact.

To lean heavily on such unobservables, each grounded in expectations formed by economic agents about some distant future, moves the problem of economic forecasting from *what is happening now* to what might be expected to happen—and leans on the forecast of the forecasts of disbursed and often disinterested economic agents. Doing so, it creates the potential for forecasting errors.

A simple policy reaction: preventing secondary acceleration

Some final observations from this simple rendering of the model are worth making. The acceleration in inflation from *t* to *t+1* can be written as:

$$\Delta\pi_{t+1} = \frac{\kappa + \beta^2}{\kappa\phi_\pi + 1}(\pi^e + r^n) - \kappa(1+\beta)\left(\frac{\kappa\phi_\pi}{\kappa\phi_\pi + 1}\right)E_{t+1}\{\hat{y}_{t+2}\} + \kappa(1+\beta)(E_{t+1}\{\hat{y}_{t+2}\} - E_t\{\hat{y}_{t+1}\})$$

Inflation in period *t+1* will further accelerate after the initial pick-up in period *t* due to an increase in inflation expectations, the natural real rate, or due to an *acceleration* in the expected output gap—though decreasing in the level of the expected output gap at time *t+2* and the policy rule response to inflation at time *t+1*.

We can ask, and answer, what value the parameter in the Taylor Rule (ϕ_π) should take to prevent inflation from accelerating (with equality) or to decelerate (with inequality):

$$\phi_\pi \geq \frac{1}{\kappa}\left[\frac{1+\beta^2/\kappa}{(1+\beta)}\frac{\pi^e + r^n}{E_t\{\hat{y}_{t+1}\}} + \left(\frac{E_{t+1}\{\hat{y}_{t+2}\} - E_t\{\hat{y}_{t+1}\}}{E_t\{\hat{y}_{t+1}\}}\right)\right]$$

The monetary policy response could be increasing in expected inflation and the natural rate relative to the previous period expectations for today's output gap. It should also be larger due an acceleration in the expected output gap or to growing inflation expectations has to be sufficiently aggressive to prevent inflation accelerating between period *t* and *t+1*.

We can therefore use this to write inflation and the output gap in period *t+1* conditional upon that policy response. Substituting this expression (with equality) into inflation at time *t+1*, as we should expect, gives:

$$\pi_{t+1} = \pi_t = \kappa(1+\beta)E_t\{\hat{y}_{t+1}\}$$

Whereas the period *t+1* output gap hinges on the prior period expectations for the output gap this period *less* the increase in inflation expectations and natural rate that occurs in *t+1* weighted by deep parameters (derivation in annex):

$$\hat{y}_{t+1} = E_t\{\hat{y}_{t+1}\} - \frac{\beta^2}{\kappa}(\pi^e + r^n)$$

or:

$$\hat{y}_t = E_t\{\hat{y}_{t+1}\} = \hat{y}_{t+1} + \frac{\beta^2}{\kappa}(\pi^e + r^n)$$

The outturn for the period *t* output gap is larger than the period *t+1* output gap by an amount that depends on the spontaneous increase in inflation expectations and natural rate at time *t+1*—if the policymaker chooses to prevent inflation from accelerating.

This comes close to the predicament facing policymakers today.

Is the period *t+1* output gap still positive? It could be. Providing the expression $\beta^2/\kappa\,(\pi^e + r^n)$ is sufficiently small, the *t+1* output gap will remain positive even if smaller than that registered the period before. However, with a large enough increase in expected inflation or the natural rate, to prevent inflation accelerating further in period *t+1*, following the initial shock in period *t*, monetary policy *may have to tighten enough to generate a negative output gap* in period *t+1*.

How strange. Even though, in this model, the boom is started by expectations of a positive output gap in period *t+1*, *ex post* the policymaker might choose to force a recession to prevent inflation accelerating—thus disappointing the very boom that began the whole inflationary shock in the first place.

Of course, we might suppose, if households anticipated this policy response, they would not expect the output boom in period *t+1* in the first place—and the inflationary impulse would never get off the ground. But this is the very reason central banks pre-committed to not respond to the initial acceleration in the first place—they wanted to pre-commit to be somewhat "irresponsible" to escape from the liquidity trap.

The fact that the acceleration in inflation may have taken them by surprise—to have "gone too far"—now means they are contemplating how to transition back to policy rules consistent with their inflation targets in this framework.

But notice how the *expected future output gap* can itself do the job of controlling inflation—in the same way that the expected output gap caused the inflation in the first period, a smaller expected second period output gap would bring inflation under control and not require as aggressive a policy response.

In any case, we are telescoping into only two periods what is a more drawn out process of adjustment through time—so should not read too much into actions at discrete moments presented here.

Consider the policy rule parameter again. Rearranging, and monetary policy would have to do nothing at all if the *t+2* output gap were expected to fall below sufficiently far below the *t+1* output gap as expected at time *t*:

$$E_{t+1}\{\hat{y}_{t+2}\} = E_t\{\hat{y}_{t+1}\} - \frac{1 + \beta^2/\kappa}{(1+\beta)}(\pi^e + r^n)$$

This is the reason why the BOE is able to signal they don't expect to hike as far as markets recently anticipated.

Annex: Period *t+1* output gap

The policy rule that delivers no acceleration in inflation at time *t+1* is given by the parameter:

$$\phi_\pi = \frac{1}{\kappa}\left[\frac{(1+\beta^2/\kappa)}{(1+\beta)}\frac{\pi^e}{E_t\{\hat{y}_{t+1}\}} + \frac{E_{t+1}\{\hat{y}_{t+2}\}}{E_t\{\hat{y}_{t+1}\}} - 1\right]$$

Or:

$$\kappa\phi_\pi + 1 = \frac{(1+\beta^2/\kappa)}{(1+\beta)}\frac{\pi^e}{E_t\{\hat{y}_{t+1}\}} + \frac{E_{t+1}\{\hat{y}_{t+2}\}}{E_t\{\hat{y}_{t+1}\}}$$

Whereas the output gap in time *t+1* is given by:

$$\hat{y}_{t+1} = \frac{1-\phi_\pi\beta^2}{\kappa\phi_\pi + 1}\pi^e + \frac{1-\kappa\phi_\pi\beta}{\kappa\phi_\pi + 1}E_{t+1}\{\hat{y}_{t+2}\}$$

Substituting in:

$$\hat{y}_{t+1} = \frac{1-\beta^2/\kappa\left[\frac{(1+\beta^2/\kappa)}{(1+\beta)}\frac{\pi^e}{E_t\{\hat{y}_{t+1}\}} + \frac{E_{t+1}\{\hat{y}_{t+2}\}}{E_t\{\hat{y}_{t+1}\}} - 1\right]}{\frac{(1+\beta^2/\kappa)}{(1+\beta)}\frac{\pi^e}{E_t\{\hat{y}_{t+1}\}} + \frac{E_{t+1}\{\hat{y}_{t+2}\}}{E_t\{\hat{y}_{t+1}\}}}\pi^e + \frac{1-\beta\left[\frac{(1+\beta^2/\kappa)}{(1+\beta)}\frac{\pi^e}{E_t\{\hat{y}_{t+1}\}} + \frac{E_{t+1}\{\hat{y}_{t+2}\}}{E_t\{\hat{y}_{t+1}\}} - 1\right]}{\frac{(1+\beta^2/\kappa)}{(1+\beta)}\frac{\pi^e}{E_t\{\hat{y}_{t+1}\}} + \frac{E_{t+1}\{\hat{y}_{t+2}\}}{E_t\{\hat{y}_{t+1}\}}}E_{t+1}\{\hat{y}_{t+2}\}$$

Rearranging as:

$$\hat{y}_{t+1} = \frac{1+\beta^2/\kappa}{\frac{(1+\beta^2/\kappa)}{(1+\beta)}\frac{\pi^e}{E_t\{\hat{y}_{t+1}\}} + \frac{E_{t+1}\{\hat{y}_{t+2}\}}{E_t\{\hat{y}_{t+1}\}} + \varepsilon}\pi^e - \beta^2/\kappa\,\pi^e + \frac{1+\beta}{\frac{(1+\beta^2/\kappa)}{(1+\beta)}\frac{\pi^e}{E_t\{\hat{y}_{t+1}\}} + \frac{E_{t+1}\{\hat{y}_{t+2}\}}{E_t\{\hat{y}_{t+1}\}} + \varepsilon}E_{t+1}\{\hat{y}_{t+2}\} - \beta E_{t+1}\{\hat{y}_{t+2}\}$$

Combining the first and third terms and simplifying gives:

$$\hat{y}_{t+1} = E_t\{\hat{y}_{t+1}\} - \frac{\beta^2}{\kappa}\pi^e$$