First draft: Nov 1st, 2021

Public finance implications of unwinding QE during a hiking cycle: an irrelevance theorem and a word of caution

The Bank of England's indemnity for the asset purchase facility could be triggered at some point now the tightening cycle has begun. The details remain secret, which raises questions why this might be the case. But in any case, should it matter for the taxpayer what strategy the BOE employ in their future balance sheet management.

More generally, with the huge amount of QE under the belts of central banks during the pandemic, and with a hiking cycle coming into view, possible central bank losses are now possible on some combination of: the flow of income (coupons below interest on reserves, IOR), realized losses (on active portfolio sales), or market to market losses (on market pricing of hikes and duration risk).

Given these issues are live, it might be useful to have a toy model to think through the difference between these losses—certainly this would help me understand the challenge facing the BOE over the period ahead to frame when bond sales make sense from a public finance perspective, which could weigh on decision-making and potentially at huge cost to the taxpayer.

And so, we here derive a (very) simple model of the impact of QE on the profit of a hypothetical central bank. And the result is quite straightforward and echoes other irrelevance theorems in macro-finance over the years.

It can be stated as follows:

> *If bonds are priced "correctly" by the term structure, given expectations of the policy rate, then the timing of bond sales is irrelevant for the net present value of monetary losses on bond portfolios. As such, the timing of bond sales is not a concern for the taxpayer.*
>
> *In this case, market to market accounting is just a way of realizing these future losses up front.*
>
> *However, in the presence of a "term" or "risk premium" on bonds, perhaps due to concern about the pace of central bank bond sales, then substantial additional—and unnecessary—losses to the taxpayer ensue.*
>
> *Thus, there is a transfer to bondholders from the taxpayer from asset sales.*

One way to interpret this is that the Bank of England's policy of selling gilts into a hiking cycle is very risky from a public finance perspective—especially if their own signally of Bank Rate is incorrect. Put another way, a policy mistake in terms of the path of Bank Rate while selling the APF portfolio could be very costly indeed.

But let's come back to this after we derive the model.

Set-up

The model is very simple and cannot here make any claim to generality. It would be interesting to explore more complicated asset holdings and coupon structures.

The model lasts four periods in total. A central bank buys a portfolio of bonds, P, at time $t = 0$ that matures at time $t = 3$. The bonds are zero coupon bonds, which simplifies the bond pricing maths considerably. We also assume that the initial policy rate is set to 0 and it expected to be unchanged until $t = 3$ when the bonds roll off. The bonds are bought at par.

The action is all in period $t = 1$ when there is a sharp change in expectations about inflation, prompting the central bank to raise interest rates—and the bond held by the APF portfolio to reprice lower given the upward revised term structure.

At this time, the central bank also has the choice to sell some part of the portfolio, μP, accumulated at $t = 0$. In which case, at time t = 3 the maturing portfolio will be equal to $(1 - \mu)P$.

This timing is shown schematically below.

Sequencing of bond purchases, sales, and market pricing

	0	1	2	3
Buy	P			
Sell		mP		(1-m)P
IOR		0	i1*(1-m)*P	i2*(1-m)P
Market yield on bond portfolio		(P/P1)^0.5 - 1	P/P2 - 1	0
Market price		P1 = P* (1/((1+i1)*(1+i2)*(1+r1)) - 1)	P2 = P* (1/(1+i2)*(1+r2)) - 1)	P

Cash flow losses over the life of the program

The interest on reserves (IOR) paid in the first period is 0 because the initial interest rate is set to zero, but we can write more generally the case flow from IOR as of time t = 1 as follows given there are no coupons on the bond portfolio:

$$IOR = -i_0 P - i_1(1 - \mu)P - i_2(1 - \mu)P$$

However, we ought to evaluate the net present value of this cash flow as of $t = 1$, evaluated using the policy rate as the risk-free rate. This is written as:

$$NPV(IOR) = -i_0 P - \frac{i_1(1-\mu)P}{(1+i_1)} - \frac{i_2(1-\mu)P}{(1+i_1)(1+i_2)}$$

Given, by assumption, $i_0 = 0$ this simplifies as

$$NPV(IOR) = -\frac{[i_1(1+i_2)+i_2]}{(1+i_1)(1+i_2)}(1 - \mu)P$$

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Realized losses

If the central bank sells some of their portfolio in period $t = 1$ they would realize losses on their portfolio.

How can we evaluate this?

The price of the portfolio P in period $t = 1$ can be written as the discounted present value of the bond given by the market yield. In general, a bond at time t = 1 can be priced by the usual bond pricing formula—the present discounted value of the coupon and principal cash flow:

$$P_1 = C + \frac{C}{(1+\hat{\imath}_1)} + \frac{P+c}{(1+\hat{\imath}_1)^2}$$

But since we are working with a zero-coupon bond, we have:

$$P_1 = \frac{P}{(1+\hat{\imath}_1)^2}$$

Where $\hat{\imath}_1$ is the market rate on this bond at time $t = 1$ which discounts the cash flow from the bond's principal, in this case two periods ahead at time $t = 3$ (hence the quadratic term).

As such, the realized loss in period 1 from selling a fraction μ of this portfolio is given by the difference between the market price and the price paid:

$$\pi = \mu(P_1 - P)$$

Substituting in the bond's price we get:

$$\pi = \mu P\left(\frac{1}{(1+\hat{\imath}_1)^2} - 1\right)$$

Suppose the bonds yield is determined by the expectations of the short rate over the next two periods plus a term premium, ρ_1. In which case: $(1+\hat{\imath}_1)^2 = (1+i_1)(1+i_2)(1+\rho_1)$ and the realized loss in the first period is given by:

$$\pi = \mu P\left(\frac{1-(1+i_1)(1+i_2)(1+\rho_1)}{(1+i_1)(1+i_2)(1+\rho_1)}\right)$$

Realized plus NPV cash flow losses

Total losses on the bond portfolio in the first period plus the NPV of future losses on monetary income due to the change in policy regime at $t = 1$ is equal to the sum of the upfront loss on portfolio sales plus the NPV of IOR losses:

$$Loss = \pi + NPV(IOR)$$

Meaning, substituting in from above, we have

$$Loss = \mu P\left(\frac{1-(1+i_1)(1+i_2)(1+\rho_1)}{(1+i_1)(1+i_2)(1+\rho_1)}\right) - \frac{[i_1(1+i_2)+i_2]}{(1+i_1)(1+i_2)}(1-\mu)P$$

Which can be simplified as:

$$Loss = \mu P\left(\frac{1-(1+\rho_1)-[i_1(1+i_2)+i_2](1+\rho_1)}{(1+i_1)(1+i_2)(1+\rho_1)}\right) - \frac{[i_1(1+i_2)+i_2](1+\rho_1)}{(1+i_1)(1+i_2)(1+\rho_1)}(1-\mu)P$$

and where the terms related to $[i_1(1+i_2)+i_2](1+\rho_1)$ that are multiplied by μ cancel in each of the first two expressions are equal and offsetting, leaving a simpler expression for the losses from the repricing of rates as of time $t = 1$:

$$Loss = -\frac{\rho_1 \mu P}{(1+i_1)(1+i_2)(1+\rho_1)} - \frac{i_1(1+i_2)+i_2}{(1+i_1)(1+i_2)}P$$

What does this mean?

Ignore the first expression on the right side for now.

The second term does not depend on either μ or the term premium in period 1, ρ_1.

The portfolio management of the APF is irrelevant when bond priced “correctly” according to the term structure. Bond sales might realize the losses up front—instead of over time. But the NPV of the cash flow saved by early bond sales is equal to the NOV loss upon sale.

As such, the timing of sales is irrelevant to the public purse in an inter-temporal sense when the NPV of the cash flow saved is discounted by the risk free rate.

Early portfolio sales realize this loss up-front. But for the public finances this doesn’t change anything.

This is analogous, I suspect, to the *Mussa Theorem* at the IMF.

This is **an irrelevance theorem for central bank bond sales**. I am not aware this is known in the macro literature—though probably is, given the various irrelevance theorems in circulation. It’s a variation on a theme—and this is hardly the first time such a situation has been pointed out.

But there is more, of course.

What if at $t = 1$ a term premium in the bond emerges, perhaps because of concern about the sale of these bonds by the central bank, such that $\rho_1 \neq 0$? Then the timing of portfolio sales matters.

If there is a positive term premium, which is often assumed to be the case, then there is an additional loss to the taxpayer from early portfolio sales given by the first term on the right above.

And the loss is increasing in the size of bond sales, μ. Put another way, if the central bank sells bonds *into an increasing term premium*, then there is a loss for the taxpayer greater than just deciding to hold these bonds to maturity.

This raises an additional challenge when a central bank, such as the BOE, wants to shrink their balance sheet. Provided bonds do not contain a term (or risk) premium, then it doesn't matter. But if the very risk of accelerated bond sales by the central bank causes such a premium, then the central bank is *unnecessarily* providing a (potentially large) lump sum transfer from the taxpayer to bondholders.

Another possibility is that the central bank signals a particular tightening path which turns out to be incorrect—which is to say the bonds reprice lower initially while the central bank is selling duration. But once it is realized that a policy mistake has been made, the bonds reprice again at a substantial initial loss to the taxpayer from selling bonds early.

(Of course, it is also possible the term premium is negative—because of the scarcity of bonds still. In which case, early sales would generate a gain to the taxpayer over the course of the QE program.)

Mark to market losses

Finally, there might be no realized losses up front from bond sales, and no initial cash flow losses in *t = 1*. But accounting rules or social convention might require the central bank be recapitalized up front for the loss when the portfolio is marked to market.

In this case, the t = 1 M2M loss can be written as:

$$M2M = P_0 - P$$

Using the bond pricing formula, whereby:

$$P_1 = \frac{P}{(1+\hat{\imath}_1)^2} = \frac{P}{(1+i_1)(1+i_2)(1+\rho_1)}$$

We again get

$$M2M = \frac{P}{(1+i_1)(1+i_2)(1+\rho_1)} - P = -\frac{\rho_1 P}{(1+i_1)(1+i_2)(1+\rho_1)} - \frac{i_1(1+i_2)+i_2}{(1+i_1)(1+i_2)} P < Loss$$

This is smaller than—meaning implies a larger loss, than that from the realized losses plus NPV of cash flow above. This is because the first term is not pre-multiplied by μ. In this case the authorities realize a loss on the entire portfolio (rather than the realized losses on initial sales alone which is greater than the cash flow loss from holding to maturity.)

If the central bank is recapped up front for this amount, then there will be a greater loss than implied by the future losses on IOR. By period 3, central bank capital will be greater by that due to the NPV of future cash flows, and the financial position of the central bank stronger than otherwise—which can, of course, be paid back later.

Alternatively, if the term premium is negative, the recap will not be enough to compensate for later cash flow losses, and smaller recaps will be needed later to compensate for the cost of the QE program.

Why does this matter?

This matters because if the rate tightening cycle happens then there will be public finance implications—rather than. I wrote about the possible cash flow losses on the BOE APF previously. My colleague Grant Wilson has written about the need for RBA recap in Australia now the rate cycle may have turned in the Financial Review there.

In addition, there is much discussion in the UK about possible losses on the BOE portfolio given the short-term nature of BOE liabilities. This is a theme taken up by the Office for Budget Responsibility, for example, in their concern about the fiscal risk from the APF and possible losses to the taxpayer.

In addition, the BOE has left open the possibility of actively selling their gilt portfolio to shrink their balance sheet early.

And the BOE-Treasury indemnity for the APF remains a mystery—will mark to market losses be realized up front, or only realized and cash flow losses?

So having a simple analytical framework to frame these issues is useful.

The fact that upfront bond sales, when bonds are correctly priced, doesn't save the taxpayer money—only realizes the losses up front—is useful. It also cautions against aggressive bank sales into a hiking cycle or the emergence of term premium on bonds.

Given the possible taxpayer losses involved, it is a pity the OBR did not spend more time on this issue instead of the cost of floating rate debt.

This note is preliminary and incomplete. Comments welcome.

Chris Marsh, London, England; November 2021